Research interests

<u>General aim</u>:

to extend the methods related to perturbation theory (PT) on the region where the coupling constant (or perturbation parameter) appears to be strong. I believe that for most problems encountered it may be achieved in two stages: (1) derivation of recurrence relations between PT coefficients which makes it possible to calculate large number of them by computer; (2) selection of suitable generalized summation procedure such as Borel method or Padé approximants in order to transform divergent PT series into convergent sequence of approximants.

In my recent work, the perturbation parameter is $1/N$, where N is the dimensionality of space. My plans for the future are related mainly to the second stage.

<u>Prehistory of my present research</u>

In my earliest papers [1, 2] the methods of PT were applied for calculating the energies and widths of bound and exited states in spherically-symmetric screened Coulomb potentials, especially Yukawa potential. We used large-order PT in powers of screening parameter.

Summation of PT series for a well-known problem of Stark effect in a hydrogen atom was considered in [4, 6, 11]. For strong field, the main problem is how to obtain *complex* energies by summation of *real* terms of PT series. We used special summation procedure, namely quadratic Padé - Hermite approximants. Convenient recurrence relations for calculation of such approximants were derived in [3].

In collaboration with V.S.Popov and co-workers from ITEP (Moscow), I was involved in 1/N-expansion for various quantum-mechanical problems, such as screened Coulomb potentials, Stark and Zeeman effects, helium-like ions, and two-center-Coulomb problem [7, 9 - 11, 13]. Somewhere we use the different term "$1/n$-expansion", because we expand the energy in powers of $1/n$, where $n = l + n_r + 1$, n_r = const, and $l \to \infty$. Our approach is equivalent to $1/N$-expansion, because we arrive to the same radial Schrödinger equation. I have calculated about 50 expansion coefficients for spherically-symmetric one-particle problems and about 10 coefficients for the case of axially symmetric or three-particle problems. Usually, Padé approximants were used to sum the series. Particularly, we were interested in quasistationary states, when an effective potential has no real minimum, so the $1/N$-expansion is complex (for example, strong-field Stark effect).

Recently we examined the asymptotics of large orders of the coefficients in $1/N$-expansion [18 - 21, 25]. Typically, they grow as factorials, $\varepsilon^{(k)} \sim C_0\, a_k\, k^\beta\, k!$ with $k \to \infty$. We found the parameters C_0, a and β by means of dispersion relations including an integral from the imaginary part of the energy. Particularly , a^{-1} equals to the action integral standing in the exponent in the quasiclassical formula for decay rate:

$$a^{-1} = \int_{r_0}^{r_1} [2(V_{\mathrm{eff}}(r) - V_0)]^{1/2}\, dr$$

where $V_0 = V_{\mathrm{eff}}(r_0)$ is the minimum of the effective potential, and r_1 is a turning point, $V_{\mathrm{eff}}(r_1) = V_0$ [19]. For bound states, there is a pair of complex-conjugate turning points, so the large-order asymptotics contains two terms: $\varepsilon^{(k)} \sim (C_0\, a_k + C_0^*\, a^*_k\, k^\beta)\, k!$ where C_0 and a are complex constants.

Details of my present and future work

In my recent research, I extend the earlier results on asymptotics of large orders of $1/N$-expansion [18 - 20] to multidimensional effective potentials for treating nonspherically-symmetric and three-particle systems. My approach to the $1/N$-expansion in large order for such systems is guided by recent studies of $1/N$-expansion for helium isoelectronic sequence, see:

Googson D.Z., López-Cabrera M. et al, J.Chem.Phys. 1992, v.97, no.11, p.8481.

This paper is concerned with calculation of the expansion coefficients to high order (~20 to 30) and with the analysis of the singularity structure of the energy as a function of $1/N$. It was shown that Padé - Borel summation incorporating results of the singularity analysis yields highly accurate energies. However, there was accounted for only the Coulombic pole at $\delta=1/N=1$. An essential singularity at $\delta=0$ responsible for the factorial growth of the expansion coefficients was found numerically, but its origin remains to be revealed. I guess that it would be very useful to know exactly the singularity of the Borel function $\delta_0=1/a$, where a is the parameter of large-order asymptotics. My object is the calculation of the parameter a for various multidimensional systems.

I convert the calculation of PT in large order into barrier-penetration problem by means of well-known dispersion techniques, see for example:

Banks T., Bender C.M., Wu T.T., Phys.Rev.D 1973, v.8, no.10, p.3346.

I deal with a multidimensional quantum decay problem (two-dimensional for axially symmetric system and three-dimensional for three-particle system).

The central problem is the solution of the eikonal equation and minimization of the classical action in order to determine the parameter a. Two different approaches are used. The first one is based on the method of characteristics. The classical trajectories in an inverted effective potential are calculated, a trajectory is chosen which terminates at a stopping point and which represents the most probable escape path, see:

A.Schmid, Ann.Phys.(N.Y.), 1986, v.170, p.333.

The parameter a equals to the reciprocal of the action along this trajectory. In the second (quite novel) approach, the action is expanded as a perturbation series around the minimum of the effective potential.

As an example that has all essential features of the general problem, I was investigating a hydrogen atom in parallel electric and magnetic fields. Just recently, two papers on this topic were prepared in collaboration with V.S.Popov [21, 25].

As a simple model, I am going to reexamine the coupled anharmonic oscillators, see:

Banks T., Bender C.M., Phys.Rev.D 1973, v.8, no.10, p.3366.

My preliminary calculations reveal some inaccuracies in Table 1 from this paper.

Also, I am going to examine a more difficult problem for helium isoelectronic sequence. Note, that there is no decay, so only complex stationary points of the classical action exist which determine the parameter $\delta_0 = a^{-1}$. I would be able to check approximate results for singularities $\delta_s \equiv \delta_0$ obtained by López-Cabrera et al in 1992. Finally, I hope to improve the convergence of Borel sums by taking into account the singularity of the Borel function $\delta_0 = 1/a$ (namely, by using Darboux approximants, accounting for square-root singularities).

At the moment, I am investigating $1/N$-expansion for near-degenerate excited states, such as |010> and |200> states of helium. For details, see enclosed abstracts, especially the second one. Here, I only notice that such degeneracy is very typical for higher excited states in multidimensional problems.