Mathematica program to check formalas

```
(* Numerical test for Model-0 *)
la=1.; (* Lambda *)
eps=10.^-10;(* small parameter used to avoid a singularity *)
V[r_]=1/8/r^2+r^2/2+la/r; (* Effective potential *)
sr0=FindRoot[r^4-la r-1/4==0,{r,1}];
r0=Replace[r,sr0[[1]]]; (* equilibrium radius *)
R=4 r0^2;A=Sqrt[1+12 r0^4];B=Sqrt[4 r0^4-1];(*variables R, A, B*)
r1=B/2/r0-r0; (*turning point*)
r2=-B/2/r0-r0; (*another turning point*)
om=2 A/R; (*frequency of vibration*)
V0=V[r0];(*minimum of the effective potential*)
V[r1]-V0(*check a turning point*)
V[r2]-V0(*check another turning point*)
Vs[r_]=D[V[r],r];Vs[r0](*check r0*)
Vss[r_]=D[Vs[r],r];Sqrt[Vss[r0]]-om(*check Omega*)
S=1/2 Log[2 r0^2 B/(1+4 r0^4+A)]+(1-12 r0^4)/(2 R)\
Log[B/(R+A)]+Pi/2 I;(*Integral of classical action*)
I1=-1/2 Log[(R+A)/(R-A)]+R/(2 A) Log[2 A^2/B/(R-B)];(* I1 *)
I2=2 Log[2 r0^2 B/(1+4 r0^4+A)]+1/r0^2 Log[(R+A)/B]+2 Pi I;(*I2*)
p[r_]=Sqrt[(r-r1)(r-r2)](r-r0)/r;(*Classical momentum*)
x=1.3;p[x]-Sqrt[2(V[x]-V0)](*Check classical momentum*)
"check for S"
S
Sr[r_]=Integrate[p[r],r];
Sr[r1+I eps]-Sr[r0+I eps]
Sr[r1-I eps]-Sr[r0-I eps]
"check for I1"
I1
I1r[r_]=Integrate[1/p[r]-1/om/(r-r0),r];
I1r[r1+I eps]-I1r[r0+I eps]
I1r[r1-I eps]-I1r[r0-I eps]
"check for I2"
I2
I2r[r_]=Integrate[(1/r^2-1/r0^2)/p[r],r];
I2r[r1+I eps]-I2r[r0+I eps]
I2r[r1-I eps]-I2r[r0-I eps]
a=1/2/S;(*parameter of large-order behavior*)
(* Printing a table of asymptotic coefficients *)
Do[Ek=N[(om/Pi/k)^(3/2) (r1-r0) Exp[om I1+I2] Re[a^(k-1/2)] k!];
Ek1=N[(om/Pi/k)^(3/2) (r1-r0) 8 A^2 B r0^4\
/(R-B)/(1+4 r0^4+A)^2 ((R-A)/(R+A))^(om/2) ((R+A)/B)^(1/r0^2)\
 Re[a^(k-1/2)] k!];
Print[k," ",Ek," ",Ek1],{k,24,25}];
```

Output of *Mathematica* program

```
          -13
3.69482 10
          -14
5.12923 10
          -14
3.10862 10
           -14
-1.66533 10
           -14
-1.58762 10
check for S
          I
1.93065 + - Pi
          2
1.93065 - 1.5708 I
1.93065 + 1.5708 I
check for I1
-0.392903
                       -7
-0.392902 + 9.20729 10   I
                       -7
-0.392902 - 9.20729 10   I
check for I2
-0.312289 + 2 I Pi
-0.312209 - 6.28311 I
-0.312209 + 6.28311 I
                        -12
24 37942. - 9.29312 10    I 37942.
                        -11
25 98738.5 - 2.4184 10    I 98738.5
```