Mathematica program to derive small-λ expansions

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(* Expansions for small Lambda *)
Clear[la];
V[r_]=1/8/r^2+r^2/2+la/r; (* Effective potential *)
r0=Sqrt[2]/2+1/2 la-Sqrt[2]/8 la^2;
R=4 r0^2;A=Sqrt[1+12 r0^4];B=Sqrt[4 r0^4-1];
r1=B/2/r0-r0; (*turning point*)
r2=-B/2/r0-r0; (*another turning point*)
om=2 A/R; (*frequency of vibration*)
V0=V[r0];(*minimum of the effective potential*)
Vs[r_]=D[V[r],r];Series[Vs[r0],{la,0,3}](*check r0*)
S=1/2 Log[2 r0^2 B/(1+4 r0^4+A)]+(1-12 r0^4)/(2 R)\
Log[B/(R+A)]+Pi/2 I;(*Integral of classical action*)
I1=-1/2 Log[(R+A)/(R-A)]+R/(2 A) Log[2 A^2/B/(R-B)];
I2=2 Log[2 r0^2 B/(1+4 r0^4+A)]+1/r0^2 Log[(R+A)/B]+2 Pi I;
a=1/2/S;(*parameter of large-order behavior*)
"Omega"
om=Normal[Simplify[Series[om,{la,0,2}]]]
"r1"
r1=Normal[Simplify[Series[r1,{la,0,2}]]]
"S"
S1=Normal[Simplify[Series[S,{la,0,2}]-Pi I/2]]
"I1"
I1=Normal[Simplify[Series[I1,{la,0,2}]-1/4 Log[la]]]
"I2"
I2=Normal[Simplify[Series[I2,{la,0,2}]]]
(*la=lamb d^(3/2);
G=Sqrt[om^3 d/Pi] (r1-r0) Exp[-2 S1/d+om I1+I2];
Simplify[Series[G,{d,0,2}]]*)
     4
O[la]
Omega
                   2
      la      11 la
2 - ------- + ------
    Sqrt[2]     8
r1
                                    3/2         2
     1        1/4            la   la           la
-(-------) + 2    Sqrt[la] - -- - ------ + ---------
  Sqrt[2]                    2       1/4   4 Sqrt[2]
                                  2 2
S
               1
la (1 - 2 Log[------] - Log[la])
                 1/4
              2 2
-------------------------------- +
            Sqrt[2]

   2             1
 la  (2 + 2 Log[------] + Log[la])
                   1/4
                2 2
 ---------------------------------
                 4
I1
             3/2         5/2          1/4
Sqrt[la]   la       23 la      Log[2 2   ] - Log[4 Sqrt[2]]
-------- - ------ - -------- + ---------------------------- +
    1/4       3/4        1/4                2
 2 2       3 2      240 2

                   1/4
 la (-6 + 2 Log[2 2   ] - Log[la])
 --------------------------------- +
             8 Sqrt[2]

   2                 1/4
 la  (31 - 18 Log[2 2   ] + 9 Log[la])
 -------------------------------------
                  64
I2
               1              1/4
2 (I Pi + Log[------] + Log[2 2   ]) +
                 1/4
              2 2

   2    11             1/4
 la  (-(--) + 4 Log[2 2   ] - 2 Log[la]) +
        2

                          1/4
 Sqrt[2] la (2 - 2 Log[2 2   ] + Log[la])
```