

# Transformation to normal modes for helium potentials using prolate spheroidal coordinates

$\lambda$	$\begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$	$\begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \end{pmatrix}$	$\begin{pmatrix}  T'_{11}  &  T'_{12}  &  T'_{13}  \\  T'_{21}  &  T'_{22}  &  T'_{23}  \\  T'_{31}  &  T'_{32}  &  T'_{33}  \end{pmatrix}$		
<b>0.1</b>	-0.03348 - 0.24337*I 0.4846 + 0.5955*I -0.2589 + 0.4367*I	1366.8 + 576.3*I -741.6 + 395.4*I -271.28 - 99.05*I	0.2503 0.397 0.889	0.1032 0.02108 0.0402	0.00660 0.1520 0.0915
<b>0.2</b>	-0.00477 - 0.27577*I 0.6051 + 0.6175*I -0.3674 + 0.4019*I	820.6 + 94.2*I -234.1 + 419.8*I -149.49 - 12.48*I	0.2856 0.382 0.858	0.1281 0.01990 0.0355	0.00647 0.1838 0.0876
<b>0.3</b>	0.02911 - 0.30448*I 0.7053 + 0.6314*I -0.4481 + 0.3693*I	517.8 - 71.0*I -20.90 + 320.60*I -80.17 + 18.57*I	0.3138 0.364 0.864	0.1500 0.01794 0.0353	0.01011 0.2157 0.0864
<b>0.4</b>	0.07048 - 0.33065*I 0.8018 + 0.6392*I -0.5150 + 0.3354*I	324.58 - 122.14*I 72.17 + 218.41*I -37.91 + 24.42*I	0.341 0.341 0.889	0.1717 0.01730 0.0367	0.01796 0.2518 0.0852
<b>0.5</b>	0.12315 - 0.35368*I 0.9043 + 0.6393*I -0.5724 + 0.2983*I	197.59 - 122.86*I 106.01 + 135.56*I -13.233 + 19.392*I	0.370 0.3098 0.928	0.1949 0.02046 0.0386	0.03047 0.2945 0.0822
<b>0.6</b>	0.19486 - 0.36964*I 1.0252 + 0.6272*I -0.6216 + 0.2553*I	114.99 - 100.24*I 110.16 + 74.97*I -0.942 + 10.797*I	0.405 0.2642 0.982	0.2216 0.02900 0.0401	0.0502 0.347 0.0750
<b>0.7</b>	0.3036 - 0.3609*I 1.1933 + 0.5862*I -0.6604 + 0.1998*I	100.92 + 33.89*I 63.30 - 66.52*I 2.4386 + 2.9332*I	0.1996 0.461 1.050	0.0428 0.2552 0.0409	0.411 0.0839 0.0606
<b>0.8</b>	0.4934 - 0.1694*I 1.5980 + 0.3340*I -0.6633 + 0.0805*I	87.54 + 5.61*I 32.738 - 17.529*I 0.15402 - 0.27477*I	0.1131 0.634 1.022	0.0581 0.2859 0.0466	0.475 0.1477 0.02803
<b>0.9</b>	2.0359 1.1322 -0.8861	67.30 61.68 0.008943	0.0613 0.1076 1.	0.2299 1.327 0.00349	1.517 0.2744 0.002793

Footnote. Here, we used variables  $s_1 = r_1$ ,  $s_2 = (r_2 + r_3) / r_1$ ,  $s_3 = (r_2 - r_3) / r_1$ . Normal coordinates are:  $q_i = \sum_j T_{ij} \Delta r_j = \sum_j T'_{ij} \Delta s_j$ , where  $T'_{ij} = \sum_k T_{ik} R_{kj}$ ,  $R_{kj} = \partial r_k / \partial s_j$ .