Transformation to normal modes for helium potential ($\lambda = 1/2$) using prolate spheroidal coordinates, with variable mass of the first electron

$1/m_1$	$\begin{pmatrix} T'_{11} & T'_{12} & T'_{13} \\ T'_{21} & T'_{22} & T'_{23} \\ T'_{31} & T'_{32} & T'_{33} \end{pmatrix}$			$\begin{pmatrix} \lvert T'_{11}\rvert & \lvert T'_{12}\rvert & \lvert T'_{13}\rvert \\ \lvert T'_{21}\rvert & \lvert T'_{22}\rvert & \lvert T'_{23}\rvert \\ \lvert T'_{31}\rvert & \lvert T'_{32}\rvert & \lvert T'_{33}\rvert \end{pmatrix}$		
1.000	-0.12791 + 0.34680*I	-0.01115 - 0.19458*I	0.026332 + 0.015322*I	0.370	0.1949	0.03047
	0.30731 - 0.03931*I	0.002190 + 0.020344*I	0.21521 - 0.20107*I	0.3098	0.02046	0.2945
	0.9166 + 0.1465*I	0.019496 - 0.033297*I	0.05699 + 0.05919*I	0.928	0.0386	0.0822
0.950	-0.11583 + 0.34271*I	-0.01212 - 0.20174*I	0.026208 + 0.011545*I	0.362	0.2021	0.02864
	0.30400 - 0.03438*I	0.003130 + 0.021496*I	0.21136 - 0.20641*I	0.3059	0.02172	0.2954
	0.9463 + 0.1356*I	0.019017 - 0.031817*I	0.05493 + 0.05604*I	0.956	0.0371	0.0785
0.900	-0.10497 + 0.33733*I	-0.01373 - 0.20894*I	0.025564 + 0.008018*I	0.353	0.2094	0.02679
	0.30240 - 0.02892*I	0.004176 + 0.022351*I	0.20729 - 0.21160*I	0.3038	0.02274	0.2962
	0.9768 + 0.1253*I	0.018683 - 0.030313*I	0.05263 + 0.05352*I	0.985	0.0356	0.0751
0.850	-0.09544 + 0.33104*I	-0.01593 - 0.21608*I	0.024490 + 0.004824*I	0.345	0.2167	0.02496
	0.30242 - 0.02328*I	0.005271 + 0.022900*I	0.20300 - 0.21662*I	0.3033	0.02350	0.2969
	1.0086 + 0.1155*I	0.018469 - 0.028809*I	0.05025 + 0.05157*I	1.015	0.0342	0.0720
0.800	-0.08724 + 0.32419*I	-0.01870 - 0.22306*I	0.023082 + 0.002013*I	0.336	0.2238	0.02317
	0.30395 - 0.01768*I	0.006365 + 0.023148*I	0.19846 - 0.22145*I	0.3045	0.02401	0.2974
	1.0425 + 0.1065*I	0.018349 - 0.027321*I	0.04788 + 0.05007*I	1.048	0.0329	0.0693
0.750	-0.08027 + 0.31707*I	-0.02196 - 0.22981*I	0.021434 - 0.000393*I	0.327	0.2309	0.02144
	0.30687 - 0.01230*I	0.007412 + 0.023104*I	0.19369 - 0.22607*I	0.3071	0.02426	0.2977
	1.0792 + 0.0981*I	0.018297 - 0.025854*I	0.04560 + 0.04895*I	1.084	0.0317	0.0669
0.700	-0.07440 + 0.30987*I	-0.02566 - 0.23628*I	0.019627 - 0.002394*I	0.319	0.2377	0.01977
	0.31108 - 0.00723*I	0.008375 + 0.022785*I	0.18870 - 0.23045*I	0.3112	0.02427	0.2978
	1.1194 + 0.0903*I	0.018288 - 0.024411*I	0.04342 + 0.04811*I	1.123	0.03050	0.0648
0.650	-0.06948 + 0.30273*I	-0.02971 - 0.24244*I	0.017729 - 0.004002*I	0.3106	0.2443	0.01818
	0.31652 - 0.00258*I	0.009220 + 0.022207*I	0.18350 - 0.23458*I	0.317	0.02404	0.2978
	1.1639 + 0.0832*I	0.018300 - 0.022987*I	0.04136 + 0.04747*I	1.167	0.02938	0.0630
0.600	-0.06537 + 0.29574*I	-0.03407 - 0.24826*I	0.015797 - 0.005239*I	0.3029	0.2506	0.01664
	0.32316 + 0.00161*I	0.009920 + 0.021390*I	0.17812 - 0.23844*I	0.323	0.02358	0.2976
	1.2140 + 0.0766*I	0.018314 - 0.021578*I	0.03941 + 0.04698*I	1.216	0.02830	0.0613
0.550	-0.06196 + 0.28896*I	-0.03867 - 0.25374*I	0.013877 - 0.006128*I	0.2955	0.2567	0.01517
	0.33100 + 0.00526*I	0.010452 + 0.020353*I	0.17257 - 0.24203*I	0.331	0.02288	0.2973
	1.2707 + 0.0704*I	0.018310 - 0.020176*I	0.03756 + 0.04656*I	1.273	0.02725	0.0598
0.500	-0.05912 + 0.28241*I	-0.04347 - 0.25888*I	0.012003 - 0.006697*I	0.2885	0.2625	0.01375
	0.34007 + 0.00828*I	0.010795 + 0.019114*I	0.16688 - 0.24535*I	0.340	0.02195	0.2967
	1.3360 + 0.0647*I	0.018271 - 0.018774*I	0.03580 + 0.04616*I	1.338	0.02620	0.0584

0.450	-0.05676 + 0.27609*I	-0.04842 - 0.26367*I	0.010206 - 0.006970*I	0.2819	0.2681	0.01236
	0.35044 + 0.01059*I	0.010930 + 0.017692*I	0.16107 - 0.24839*I	0.351	0.02080	0.2960
	1.4122 + 0.0593*I	0.018176 - 0.017364*I	0.03411 + 0.04573*I	1.413	0.02514	0.0570
0.400	-0.05482 + 0.27001*I	-0.05347 - 0.26811*I	0.008508 - 0.006975*I	0.2755	0.2734	0.01100
	0.36221 + 0.01206*I	0.010841 + 0.016106*I	0.15514 - 0.25115*I	0.362	0.01941	0.2952
	1.5026 + 0.0542*I	0.018003 - 0.015938*I	0.03247 + 0.04520*I	1.504	0.02404	0.0556
0.350	-0.05322 + 0.26415*I	-0.05860 - 0.27222*I	0.006927 - 0.006733*I	0.2695	0.2785	0.00966
	0.37548 + 0.01248*I	0.010512 + 0.014374*I	0.14911 - 0.25365*I	0.376	0.01781	0.2942
	1.6121 + 0.0495*I	0.017723 - 0.014489*I	0.03084 + 0.04449*I	1.613	0.02289	0.0541
0.2500	-0.05084 + 0.25304*I	-0.06895 - 0.27945*I	0.004170 - 0.005608*I	0.2581	0.2878	0.00699
	0.40723 + 0.00917*I	0.009067 + 0.010546*I	0.13677 - 0.25789*I	0.407	0.01391	0.2919
	1.9245 + 0.0408*I	0.016687 - 0.011482*I	0.02746 + 0.04210*I	1.925	0.02026	0.0503
0.2000	-0.04998 + 0.24777*I	-0.07410 - 0.28259*I	0.003015 - 0.004766*I	0.2528	0.2921	0.00564
	0.42609 + 0.00461*I	0.007918 + 0.008492*I	0.13045 - 0.25966*I	0.426	0.01161	0.2906
	2.1636 + 0.0367*I	0.015806 - 0.009899*I	0.02555 + 0.04007*I	2.164	0.01865	0.0475
0.1500	-0.04929 + 0.24266*I	-0.07921 - 0.28542*I	0.002017 - 0.003766*I	0.2476	0.2962	0.00427
	-0.4472 + 0.0027*I	-0.006459 - 0.006375*I	-0.12402 + 0.26121*I	0.447	0.00908	0.2892
	2.5144 + 0.0325*I	0.014533 - 0.008233*I	0.023270 + 0.037056*I	2.515	0.01670	0.0438
0.1000	-0.04875 + 0.23769*I	-0.08423 - 0.28795*I	0.0011810- 0.002626*I	0.2426	0.3000	0.002880
	-0.4708 + 0.0137*I	-0.004670 - 0.004225*I	-0.11743 + 0.26256*I	0.471	0.00630	0.2876
	3.1027 + 0.0278*I	0.012637 - 0.006431*I	0.020256 + 0.032420*I	3.103	0.01418	0.0382
0.0500	-0.04832 + 0.23286*I	-0.08916 - 0.29019*I	0.000508 - 0.001365*I	0.2378	0.3036	0.001457
	-0.4969 + 0.0298*I	-0.0025258 - 0.0020817*I	-0.11061 + 0.26373*I	0.498	0.00327	0.2860
	4.4266 + 0.0213*I	0.009550 - 0.004335*I	0.015521 + 0.024635*I	4.43	0.01049	0.02912
0.03000	-0.04818 + 0.23097*I	-0.09109 - 0.29101*I	0.0002854- 0.000831*I	0.2359	0.3049	0.000878
	-0.5079 + 0.0381*I	-0.0015627 - 0.0012378*I	-0.10780 + 0.26414*I	0.509	0.001994	0.2853
	5.737 + 0.017*I	0.007606 - 0.003293*I	0.012481 + 0.019651*I	5.74	0.00829	0.02328
0.02000	-0.04812 + 0.23002*I	-0.09205 - 0.29140*I	0.000184 - 0.000558*I	0.2350	0.3056	0.000587
	-0.5136 + 0.0427*I	-0.0010577 - 0.0008208*I	-0.10637 + 0.26433*I	0.515	0.001339	0.2849
	7.041 + 0.014*I	0.006299 - 0.002662*I	0.010395 + 0.016283*I	7.04	0.00684	0.01932
0.01000	-0.04806 + 0.22908*I	-0.09300 - 0.29178*I	0.000089 - 0.000281*I	0.2341	0.3062	0.000294
	-0.5193 + 0.0476*I	-0.0005369 - 0.0004080*I	-0.10492 + 0.26452*I	0.521	0.000674	0.2846
	9.978 + 0.010*I	0.004518 - 0.001864*I	0.007505 + 0.011685*I	9.98	0.00489	0.01389

Footnote. Here, we used variables $s_1 = r_1$, $s_2 = (r_2 + r_3)/r_1$, $s_3 = (r_2 - r_3)/r_1$. Normal coordinates are: $q_i = \sum_j T_{ij} \Delta r_j = \sum_j T'_{ij} \Delta s_j$, where $T'_{ij} = \sum_k T_{ik} R_{kj}$, $R_{kj} = \partial r_k / \partial s_j$. Results imply that the slowest normal-mode coordinate q_3 approximately equals to s_1, and coordinates q_1, q_2 equal to prolate spheroidal coordinates s_2, s_3 with mixing of s_1.